

Algorithm for Improving the Computing Power of Next Generation Wireless Receivers

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Abstract

Next generation wireless receivers demand low computational complexity algorithms with high computing power in order to perform fast signal detections and error estimations. Several signal detection and estimation algorithms have been proposed for next generation wireless receivers which are primarily designed to provide reasonable performance in terms of signal to noise ratio (SNR) and bit error rate (BER). However, none of them have been chosen for direct implementation as they offer high computational complexity with relatively lower computing power. This paper presents a low-complexity power-efficient algorithm that improves the computing power and provides relatively faster signal detection for next generation wireless multiuser receivers. Measurement results of the proposed algorithm are provided and the overall system performance is indicated by BER and the computational complexity. Finally, in order to verify the low-complexity of the proposed algorithm we also present a formal mathematical proof.

Category: Smart and intelligent computing

Keywords: Computing power; DS-CDMA systems; Power efficiency; Wireless communications; Wireless receive

I. INTRODUCTION

Code division multiple access (CDMA) has been widely used and accepted for wireless access in terrestrial and satellite applications. CDMA cellular systems use state of the art digital communication techniques and build on some of the most sophisticated aspects of the modern statistical communication theory. The CDMA technique has significant advantages over the analog and conventional time division multiple access (TDMA) system. CDMA is a multiple access (MA) technique that uses spread spectrum modulation where each user has its own unique chip sequence. This technique enables multiple users to access a common channel simultaneously.

Multiuser direct sequence (DC)-CDMA has received wide

attention in the field of wireless communications [1, 2]. In CDMA communication systems, several users are active on the same fringe of the spectrum at the same time. Therefore, the received signal results from the sum of all the contributions from the active users [3]. Conventional spread spectrum mechanisms applied in DS-CDMA are severely limited in performance by multiple access interference (MAI) [1, 4], leading to both system capacity limitations and strict power control requirements. The traditional way to deal with such a situation would be to process the received signal through parallel devices.

Verdu [5] proposed and analyzed the optimum multiuser detector and the maximum likelihood (ML) sequence detector, which, unfortunately, is too complex for practical implementation, since its complexity grows exponen-

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tially as the function of the number of users. Although the performance of the multiuser detector is optimum, it is not a very practical system since the number of required computations increases by $2k$, where k is the number of users to be detected. Multiuser detectors suffer from their relatively higher computational complexity that prevents CDMA systems from adapting this technology for signal detection. However, if we could lower the complexity of the multiuser detectors, most of the CDMA systems would likely take advantage of this technique in terms of its increased computing power and superior data rate.

In this paper, we employ a new scheme that observes the coordinates of the constellation diagram to determine the location of the transformation points. Since most of the decisions are correct, we can reduce the number of the required computations by only using the transformation matrices on those coordinates which are most likely to lead to an incorrect decision. By doing this, we can greatly reduce the unnecessary processing involved in making decisions concerning the correct regions or the coordinates. Our mathematical results show that the proposed approach successfully reduces the computational complexity of the optimal ML receiver.

The rest of this paper is organized as follows. Section II describes the state of the art research that has already been done in this area. Section III presents both the original ML algorithm and the proposed power efficient (PE) algorithm along with a comprehensive discussion of their computational complexities. The numerical and simulation results are presented in Section IV. Finally, we conclude the paper in Section V.

II. RELATED WORK

Multiuser receivers can be categorized in the following two forms: optimal maximum likelihood sequence estimation (MLSE) receivers and suboptimal linear and non-linear receivers. Suboptimal multiuser detection algorithms can be further classified into linear and interference cancellation type algorithms. The figurative representation of the research work that has been done so far in this area is presented in Fig. 1. The optimal multiuser wireless receiver consists of a matched filter followed by an ML sequence detector implemented via a dynamic programming algorithm. In order to mitigate the problem of MAI, Verdu [6] proposed and analyzed the optimum multiuser detector for asynchronous Gaussian MA channels. The optimum detector searches for all the possible demodulated bits in order to locate the decision region that maximizes the correlation metric given by [5]. The practical application of this mechanism is limited by the complexity of the receiver [7]. This optimum detector outperforms the conventional detector, but unfortunately its complexity grows exponentially in the order of $O(2)^K$, where K is the number of active users.

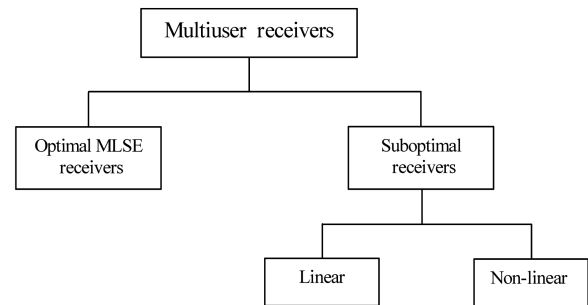


Fig. 1. Multiuser optimal and suboptimal wireless receivers with linear and non-linear detectors. MLSE: maximum likelihood sequence estimation.

Much research has been done to reduce the computational complexity of this receiver. Agrell and Ottosson [8] proposed a new ML receiver that uses the neighbor descent (ND) algorithm. They implemented an iterative approach using the ND algorithm to locate the region where the actual observations belong. In order to reduce the computational complexity of optimum receivers, the iterative approach uses the ND algorithm that performs MAI cancellation linearly. The linearity of their iterative approach increases noise components at the receiving end. Due to the enhancement in the noise components, the signal to noise ratio (SNR) and bit error rate (BER) of the ND algorithm are more affected by the MAI.

Several tree-search detection receivers have been proposed in the literature [9, 10], to reduce the computational complexity of the original ML detection scheme proposed by Verdu. Specifically, [9] investigated a tree-search detection algorithm, where a recursive, additive metric was developed in order to reduce the search complexity. Reduced tree-search algorithms, such as the well-known M- and T-algorithms [11], were used by [10] to reduce the complexity incurred by the optimum multiuser detectors.

To make an optimal wireless receiver that provides minimum mean square error (MMSE) performance, we need to provide some knowledge of interference such as phase, frequency, delays, and amplitude for all users. In addition, an optimal MMSE receiver requires the inversion of a large matrix. This computation takes a relatively long time and makes the detection process slow and expensive [2, 7]. On the other hand, an adaptive MMSE receiver greatly reduces the entire computation process and gives an acceptable performance. Xie et al. [12] proposed an approximate MLSE solution known as the pre-survivor processing (PSP) type algorithm, which combined a tree search algorithm for data detection with the aid of the recursive least square (RLS) adaptive algorithm used for channel amplitude and phase estimation. The PSP algorithm was first proposed by Seshadri [13] for a blind estimation in single user inter-symbol interference (ISI)-contaminated channels.

III. PROPOSED LOW-COMPLEXITY PE ALGORITHM

We consider a synchronous DS-CDMA system as a linear time invariant (LTI) channel. In an LTI channel, the probability of variations in the interference parameters, such as the timing of all users, amplitude variation, phase shift, and frequency shift, is extremely low. This property makes it possible to reduce the overall computational complexity at the receiving end. Our PE algorithm utilizes the complex properties of the existing inverse matrix algorithms to construct the transformation matrices and to determine the location of the transformation points that may occur in any coordinate of the constellation diagram. The individual transformation points can be used to determine the average computational complexity.

The system may consist of k users. User k can transmit a signal at any given time with the power of W_k . With the binary phase shift keying (BPSK) modulation technique, the transmitted bits belong to either +1 or -1, (i.e., $b_k \in \{\pm 1\}$).

The cross correlation can be reduced by neglecting the variable delay spreads, since these delays are relatively small as compared to the symbol transmission time. To detect signals from any user, the demodulated output of the low pass filter is multiplied by a unique signature waveform assigned by a pseudo-random number generator. It should be noted that we extract the signal using the match filter followed by a Viterbi algorithm.

A. Original Optimum Multiuser (ML) Receiver

The optimum multiuser receiver exists and permits to relax the constraints of choosing the spreading sequences with good correlation properties at a cost of increased receiver complexity. Fig. 2 shows the block diagram of an optimum receiver that uses a bank of matched filters and an ML Viterbi decision algorithm for signal detection. It should be noted in Fig. 2 that the proposed PE algorithm is implemented in conjunction with the Viterbi decision algorithm with the feedback mechanism. To detect a signal from any user, the demodulated output of the low pass filter is multiplied by a unique signature waveform assigned by a pseudo-random number generator.

When the receiver wants to detect the signal from user-1, it first demodulates the received signal to obtain the base-band signal. The base-band signal multiplies with user-1's unique signature waveform, $c_1(t)$. The resulting signal, $r_1(t)$, is applied to the input of the matched filter. The matched filter integrates the resulting signal $\{r_1(t)\}$ over each symbol period T , and the output is read into the decoder at the end of each integration cycle. The outputs of the matched filter and the Verdu's algorithm [5] can be represented by $y_k(m)$ and $b_k(m)$, respectively, where m is the sampling interval. We also assume that the first timing offset τ_1 is almost zero and $\tau_2 < T$. The same procedure applies to other users. The outputs of the matched

filter for the first two users at the m^{th} sampling interval can be expressed as follows:

$$y_1(m) = \frac{1}{T} \left\{ \int_m^{(m+1)T} r_1(t) C_1(t) dt \right\} \quad (1)$$

$$y_2(m) = \frac{1}{T} \left\{ \int_{2+(m)T}^{2+(m+1)T} r_2(t) C_2(t - \tau_2) dt \right\} \quad (2)$$

The received signals $r_1(t)$ and $r_2(t)$ can be extracted from the first two equations as follows:

$$r_1(t) = (E_{C1})^{0.5} \sum_{i=-M}^M b_1(i) C_1(t - iT_b) \quad (3)$$

$$r_2(t) = (E_{C2})^{0.5} \sum_{i=-M}^M b_2(i) C_2(t - iT_b - \tau_2) \quad (4)$$

where E_{C1} and E_{C2} represent the original bit energy of the received signals with respect to their unique signature waveforms.

The received signals $r_1(t)$ and $r_2(t)$ can be combined in one signal $r(t)$ that will be distinguished by the receiver with respect to its unique signature waveform. Based on the above analysis, we can combine Equations (3) and (4).

$$r(t) = (E_{C1})^{0.5} \sum_{i=-M}^M b_1(i) C_1(t - iT_b) + (E_{C2})^{0.5} \sum_{i=-M}^M b_2(i) C_2(t - iT_b - \tau_2) \quad (5)$$

Substitute (5) as an individual equation into (1), and we have

$$y_1(m) = \frac{1}{T} \left\{ \int_m^{(m+1)T} \left\{ (E_{C1})^{0.5} \sum_{i=-M}^M b_1(i) \right\} \left\{ C_1(t - iT_b) \right\} dt \right\} \quad (6)$$

Substitute (5) as an individual equation into (2), and we have

$$y_2(m) = \frac{1}{T} \left\{ \int_{2+(m)T}^{2+(m+1)T} \left\{ (E_{C2})^{0.5} \sum_{i=-M}^M b_2(i) \right\} \left\{ C_2(t - iT_b - \tau_2) \right\} dt \right\} \quad (7)$$

After performing integration over the given interval, we obtain the following results with the noise components as well as the cross correlation of signature waveforms.

$$y_1(m) = (E_{C1})^{0.5} b_1(m) + (E_{C2})^{0.5} b_2(m-1) \rho_1 + (E_{C2})^{0.5} b_2(m) \rho_0 + (E_{C2})^{0.5} b_2(m+1) \rho_{-1} + n_1(m) \quad (8)$$

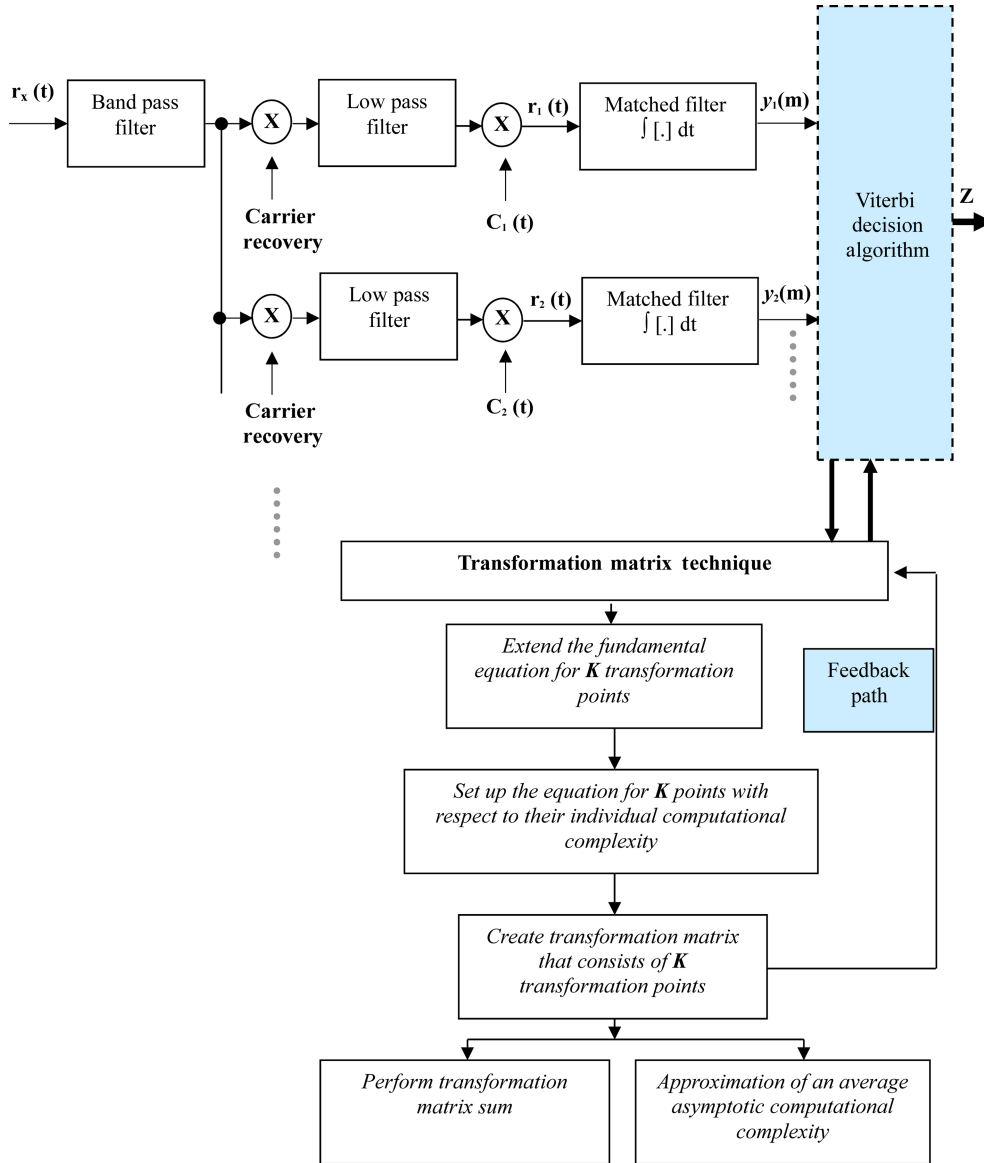


Fig. 2. Implementation of the proposed transformation matrix algorithm with the optimum multiuser receiver.

$$y_2(m) = (E_{C_2})^{0.5} b_2(m) + (E_{C_1})^{0.5} b_1(m-1) \rho_1 + (E_{C_1})^{0.5} b_1(m) \rho_0 + (E_{C_1})^{0.5} b_1(m+1) \rho_{-1} + n_2(m) \quad (9)$$

where coefficients $b_1(m)$ and $b_2(m)$ represent MAI, $\rho_{-1/0/+1}$ are cross-correlations of signature waveforms, and $n_1(m)$ and $n_2(m)$ represent the minimum noise components. Since the channel is LTI, the probability of unwanted noise is minimal.

These symbols can now be decoded using an ML Viterbi decision algorithm. Viterbi algorithm can be used to detect these signals in much the same way as convolution codes. This algorithm makes a decision over a finite window of sampling instants rather than waiting for all the data to be received [1]. The above derivation can be extended

from two users to K number of users. The number of operations performed in the Viterbi algorithm is proportional to the number of decision states, and the number of decision states is exponential with respect to the total number of users. The asymptotic computational complexity of this algorithm can be approximated as: $O(2)^K$

B. Proposed PE Algorithm

According to original Verdu's algorithm [5], the outputs of the matched filter $y_1(m)$, and $y_2(m)$ can be considered as a single output $y(m)$. In order to minimize the noise components and to maximize the received demodulated bits, the output of the matched filter can be transformed, and this transformation can be expressed as

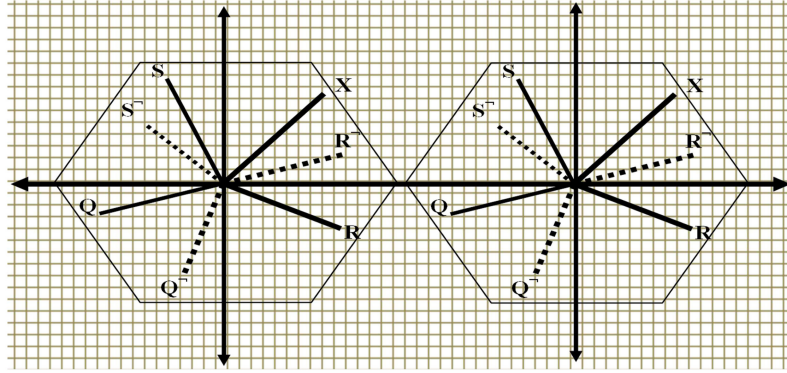


Fig. 3. A constellation diagram consisting of three different vectors. $Q, R,$ and S represent vectors or transformation points within the coverage area of a cellular network. $Q^{\bar{}}, R^{\bar{}}$, and $S^{\bar{}}$ represent the computational complexity of each individual transformation point.

follows: $y(m) = Tb + \eta$ where T represents the transformation matrix, and $b_k \in \{\pm 1\}$ and η represent the noise components.

In addition, if vectors are regarded as points in a K -dimensional space, then the vectors constitute the constellation diagram that has K total points. This constellation diagram can be mathematically expressed as: $\aleph = \{Tb\}$ where $b \in \{-1, +1\}$. We use this equation as a fundamental equation of the proposed algorithm. According to the detection rule, the constellation diagram can be partitioned into 2^K lines (where the total possible lines in the constellation diagram can be represented as f) that can only intersect each other at the following points: $\aleph = \{Tb\}_{b \in \{-1, 1\}}^K \setminus f$

Fig. 3 illustrates the constellation diagram that consists of three different vectors (lines) with the original vector ‘ X ’ that represents the collective complexity of the receiver. $Q, R,$ and S represent vectors or transformation points within the coverage area of a cellular network as shown in Fig. 3. In addition, $Q^{\bar{}}, R^{\bar{}}$, and $S^{\bar{}}$ represent the computational complexity of each individual transformation point. To compute the collective computational complexity of the optimum wireless receiver, it is essential to determine the complexity of each individual transformation point. The computational complexity of each individual transformation point is represented by $X^{\bar{}}$ which is equal to the collective complexity of $Q^{\bar{}}, R^{\bar{}}$, and $S^{\bar{}}$. A transformation matrix defines how to map points from one coordinate space into another. A transformation does not change the original vector, instead it alters the components. To derive the value of the original vector X , we need to perform the following derivations. We consider the original vector with respect to each transmitted symbol or bit. The following system can be derived from the above equations:

$$\begin{pmatrix} X^{\bar{}}Q \\ X^{\bar{}}R \\ X^{\bar{}}S \end{pmatrix} = \begin{pmatrix} ii^{\bar{}} & ji^{\bar{}} & ki^{\bar{}} \\ ij^{\bar{}} & jj^{\bar{}} & kj^{\bar{}} \\ ik^{\bar{}} & jk^{\bar{}} & kk^{\bar{}} \end{pmatrix} \begin{pmatrix} XQ \\ XR \\ XS \end{pmatrix} \quad (10)$$

Equation (10) represents the following: QRS with the unit vectors $i, j,$ and k ; $X^{\bar{}}Q, X^{\bar{}}R,$ and $X^{\bar{}}S$ with the inverse of the unit vectors $i^{\bar{}}, j^{\bar{}}$, and $k^{\bar{}}$. The second matrix on the right hand side of (10) represents b , whereas the first matrix on the right hand side of (10) represents the actual transformation matrix. Therefore, the transformation matrix from the global reference points (which could be $Q, R,$ or S) to a particular local reference point can now be derived from (10):

$$\begin{pmatrix} X^{\bar{}}Q \\ X^{\bar{}}R \\ X^{\bar{}}S \end{pmatrix} = T_{LG} \begin{pmatrix} XQ \\ XR \\ XS \end{pmatrix} \quad (11)$$

Equation (11) can also be written as:

$$T_{LG} = \begin{pmatrix} ii^{\bar{}} & ji^{\bar{}} & ki^{\bar{}} \\ ij^{\bar{}} & jj^{\bar{}} & kj^{\bar{}} \\ ik^{\bar{}} & jk^{\bar{}} & kk^{\bar{}} \end{pmatrix} \quad (12)$$

In Equation (12), the dot products of the unit vectors for the two reference points are in fact the same as the unit vectors of the inverse transformation matrix of (11). We need to compute the locations of the actual transformation points described in Equations (11) and (12). Let the unit vectors for the local reference point be:

$$\begin{aligned} i^{\bar{}} &= [t_{11}(i), t_{12}(j), t_{13}(k)] \\ j^{\bar{}} &= [t_{21}(i), t_{22}(j), t_{23}(k)] \\ k^{\bar{}} &= [t_{31}(i), t_{32}(j), t_{33}(k)] \end{aligned} \quad (13)$$

Since, $i^{\bar{}}(i+j+k) = i^{\bar{}}$ where $(i+j+k) = 1$. The same is true for the rest of the unit vectors ($i^{\bar{}} = i^{\bar{}}$). Therefore, (13) can be rewritten as:

$$\begin{aligned} i^{\bar{}} &= [t_{11}, t_{12}, t_{13}] \\ j^{\bar{}} &= [t_{21}, t_{22}, t_{23}] \\ k^{\bar{}} &= [t_{31}, t_{32}, t_{33}] \end{aligned} \quad (14)$$

By substituting the values of i^- , j^- , and k^- from (14) into (12), we obtain

$$T_{UG} = \begin{pmatrix} i(t_{11}(i), t_{12}(j), t_{13}(k)) & j(t_{11}(i), t_{12}(j), t_{13}(k)) & k(t_{11}(i), t_{12}(j), t_{13}(k)) \\ i(t_{21}(i), t_{22}(j), t_{23}(k)) & j(t_{21}(i), t_{22}(j), t_{23}(k)) & k(t_{21}(i), t_{22}(j), t_{23}(k)) \\ i(t_{31}(i), t_{32}(j), t_{33}(k)) & j(t_{31}(i), t_{32}(j), t_{33}(k)) & k(t_{31}(i), t_{32}(j), t_{33}(k)) \end{pmatrix} \begin{pmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{pmatrix} \quad (15)$$

Substituting T_{UG} from (15) into (11), yields

$$\begin{pmatrix} X^-Q \\ X^-R \\ X^-S \end{pmatrix} = \begin{pmatrix} t_{11} & t_{12} & t_{13} \\ t_{21} & t_{22} & t_{23} \\ t_{31} & t_{32} & t_{33} \end{pmatrix} \begin{pmatrix} XQ \\ XR \\ XS \end{pmatrix} \quad (16)$$

Equation (16) corresponds to the following standard equation used for computing the computational complexity at the receiving end: $\aleph = \{Tb\}_{b \in \{-1, +1\}^k}$. If the target of one transformation ($U:Q \rightarrow R$) is the same as the source of another transformation ($T:R \rightarrow S$), then we can combine two or more transformations and form the following composition: $TU:Q \rightarrow S$, $TU(Q) = T(U(Q))$.

This composition can be used to derive the collective computational complexity at the receiving end using (16). Since the channel is assumed to be LTI, the transformation points may occur in any coordinate of the constellation diagram. The positive and negative coordinates of the constellation diagram do not make any difference for an LTI propagation channel. In addition, the transformation points should lie within the specified range of the system. Since we assumed that the transmitted signals are modulated using BPSK which can at most use 1 bit out of 2 bits (that is, $b_k \in \{\pm 1\}$), consider the following set of transformation points to approximate the number of demodulated received bits that need to search out by decision algorithm:

$$\aleph = \left((1 \ 1 \ 1) \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{pmatrix} + (1 \ 1 \ 1) \begin{pmatrix} -1 & 1 & 1 \\ -1 & 1 & 1 \\ -1 & 1 & 1 \end{pmatrix} \right)^k \\ + \left((1 \ -1 \ 1) \begin{pmatrix} 1 & -1 & -1 \\ -1 & 1 & -1 \\ -1 & -1 & 1 \end{pmatrix} + (1 \ -1 \ 1) \begin{pmatrix} -1 & -1 & -1 \\ -1 & -1 & -1 \\ -1 & -1 & -1 \end{pmatrix} \right)^k$$

Using (16), a simple matrix addition of the received demodulated bits can be used to approximate the number of most correlated transformation points. The set of the transformation points correspond to the actual location within the transformation matrix as shown in (16).

C. Complexity of the PE Algorithm

The entire procedure for computing the number of

demodulated bits that need to be searched by the decision algorithm can be used to approximate the number of most correlated signals for any given set of transformation points. This is because, we need to check whether or not the transformation points are closest to either (+1, +1) or (-1, -1). The decision regions or the coordinates where the transformation points lie for (+1, +1) and (-1, -1) are simply the corresponding transformation matrixes that store the patterns of their occurrences. If the transformation points do not exist in the region (coordinate) of either (+1, +1) or (-1, -1), then it is just a matter of checking whether the transformation points are closest to (+1, -1) or to (-1, +1). In other words, the second matrix on the right hand side of (16) requires a comprehensive search of at most 5^k demodulated bits that indirectly correspond to one or more users.

The minimum search performed by the decision algorithm is conducted if the transformation points exist within the incorrect region. Since the minimum search saves computation by one degree, the decision algorithm has to search at least 4^k demodulated bits. The average number of computations required by a system on any given set always exists between the maximum and the minimum number of computations performed in each operational cycle [14]. This implies that the total number of demodulated bits that need to be searched by the decision algorithm cannot exceed $5^k - 4^k$. In other words, the total numbers of most correlated pairs are upper-bounded by $5^k - 4^k$.

Since most of the decisions are correct, we can reduce the number of computations by only using the transformation matrixes on those coordinates that are most likely to lead to an incorrect decision. By doing this, we greatly reduce the unnecessary processing required to make a decision regarding the correct region or the coordinate. Thus, the number of received demodulated bits that need to be searched can be approximated as: $5^k - 4^k$. The total numbers of pairs in the upper-bound describe the computational complexity at the receiving end.

The computational complexity of any multiuser receiver can be quantified by its time complexity per bit [14]. The collective computational complexity of the proposed PE algorithm is achieved after performing the transformation matrix sum using the complex properties of the existing inverse matrix algorithms. In other words, the computational complexity can be computed by determining the number of operations required by the receiver to detect and demodulate the transmitted information divided by the total number of demodulated bits. Therefore, both quantities T and b from our fundamental equation can be computed together and the generation for all the values of demodulated received bits b can be done through the sum of the actual T that approximately takes $O(5/4)^k$ operations with an asymptotic constant. We determine the collective complexity of the optimum multiuser receiver by performing the transformation matrix sum.

Once the BPSK modulated bits ($b_k \in \{\pm 1\}$) and the transformation points are determined, the collective complexity of the optimal ML receiver can be approximated by performing the sum of transformation matrices. The resultant approximation by the decision algorithm can only be used to analyze the number of operations performed by the receiver. The complexity of the proposed algorithm is not polynomial in the number of users, instead the number of operations required to maximize the demodulation of the transmitted bits and to choose an optimal value of b is bounded by $O(5/4)^K$, and therefore the time complexity per bit is $O(5/4)^K$. Even though the computational complexity of the proposed algorithm is not polynomial in terms of the total number of users it still provides significantly reduced computational complexity.

D. Mathematical Proofs for Computational Complexity

This section provides the formal mathematical proof of the above discussion that proves the efficiency of the proposed algorithm with given input sizes. We provide a mathematical proof for both the upper bound and the lower bound of the proposed algorithm over the ND and the ML algorithms.

Proof (1). $f(x)$ is upper bound of $g_1(x)$ and $g_2(x)$

For the sake of this proof, we consider that each algorithm is represented by the growth of a function as follows: Let $f(x) = (5/4)^K$ for the proposed algorithm, $g_1(x) = (2)^K$ for the ML algorithm [5], and $g_2(x) = (3/2)^K$ for the ND algorithm [2]. It should be noted that $O(5/4)$ is the derived complexity of the proposed algorithm as shown in the previous section.

Equation (17) shows that the proposed algorithm $f(x)$ is in the lower bound of both $g_1(x)$ and $g_2(x)$. Therefore, the values of the function $f(x)$, with different input sizes, always exist as a lower limit of both $g_1(x)$ and $g_2(x)$. To prove this hypothesis mathematically, we need to consider the following equations:

$$f(x) = O(g_1(x)) \text{ and } f(x) = O(g_2(x)) \tag{17}$$

$$\begin{aligned} f(x) &= (5/4)^K < c_1(g_1(x)), \\ f(x) &= (5/4)^K < c_2(g_2(x)) \end{aligned}$$

Solving for $g(x)$, we acquire the following two equations:

$$f(x) = (5/4)^K < c_1(2.0)^K \tag{18}$$

$$f(x) = (5/4)^K < c_2(3/2)^K \tag{19}$$

Solving for $g_1(x)$, we can write an argument using (18), such as: $f(x)$ is said to be $O(c_1 \times g_1(x))$, if and only if there exists a constant c_1 and the threshold n_o such that:

$$f(x) < c_1|g(x)| \text{ whenever } x > n_o,$$

$$f(x) = O(c_1 \times g(x))$$

Thus, this is proved using (18). It should be noted that the n_o is the threshold value at which both functions approximately approach each other. Solving for $g_2(x)$, we can write a similar argument using (19), such as:

$$f(x) \text{ is said to be } O(c_2 \times g_2(x)),$$

if and only if there exists a constant c_2 and the threshold n_o such that:

$$f(x) < c_2|g(x)| \text{ Whenever } x > n_o,$$

$$f(x) = O(c_2 \times g(x))$$

Thus, this is proved using (19).

Proof (2). $f(x)$ is lower bound of $g_1(x)$ and $g_2(x)$

To analyze the lower bound, we provide a proof in the reverse order to define a lower bound for the function $f(x)$. Equation (20) demonstrates that both functions $g_1(x)$ and $g_2(x)$ are the upper bounds for the function $f(x)$. The corresponding values of $g_1(x)$ and $g_2(x)$ with different input sizes always lie as a maximum upper limit of $f(x)$, and hence both functions $g_1(x)$ and $g_2(x)$ always yield a greater complexity. To prove this hypothesis mathematically, we need to consider the following equations:

$$\begin{aligned} g_1(x) &= \Omega(f(x)), \\ g_2(x) &= \Omega(f(x)) \end{aligned} \tag{20}$$

$$\begin{aligned} g_1(x) &= (2.0)^K > c_1(f(x)) \\ g_2(x) &= (3/2)^K > c_1(f(x)) \end{aligned}$$

Solving for $f(x)$, we acquire the following two equations:

$$g_1(x) = (2.0)^K > c_1(5/4)^K \tag{21}$$

$$g_2(x) = (3/2)^K > c_2(5/4)^K \tag{22}$$

Solving for $g_1(x)$, we can make the following argument using (21), such as

$$g_1(x) \text{ is said to be } \Omega(c_1 \times f(x)),$$

if and only if there exists a constant c_1 and the threshold n_o such that:

$$g_1(x) < c_1|f(x)| \text{ whenever } x > n_o,$$

$$g_1(x) = \Omega(c_1 \times f(x))$$

Thus, this is proved using (21). Solving for $g_2(x)$, we can claim a similar argument using (22), such as $g_2(x)$ is

said to be $\Omega(c_2 \times f(x))$, if and only if there exists a constant c_2 and the threshold n_o such that:

$$g_2(x) > c_2|f(x)|$$

$$g_2(x) = \Omega(c_2 \times f(x))$$

Thus, this is proved using (22). As we have proved here (referring (17) and (20)) that:

$$f(x) = \mathcal{O}(c \times g(x)) \text{ and } g(x) = \Omega(c \times f(x))$$

IV. Performance Analysis of the PE Algorithm

The order of growth of a function is an important criterion for proving the complexity and efficiency of an algorithm. It provides simple characterization of the algorithm's efficiency and also allows us to compare the relative performance of algorithms with given input sizes. In this section, numerical analysis is performed to observe the computational complexity of the proposed algorithm. Simulations are conducted in MATLAB (MathWorks, Natick, MA, USA) to evaluate the BER performance of a time-invariant DS-CDMA linear synchronous system in an additive white Gaussian noise (AWGN) channel. The original computational complexity of the ML optimal receiver is $(2)^K$ [5]. Another research paper [8] has reduced the complexity from $(2)^K$ to $(3/2)^K$. This paper [8], also known as the ND algorithm, has reduced the computational complexity after considering a synchronous DS-CDMA system.

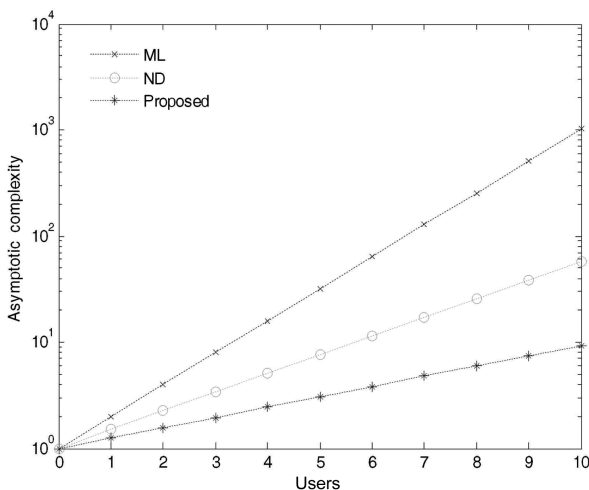


Fig. 4. The asymptotic computational complexities versus number of users for a small-capacity network ($0 < K \leq 10$). ML: maximum likelihood, ND: neighbor descent.

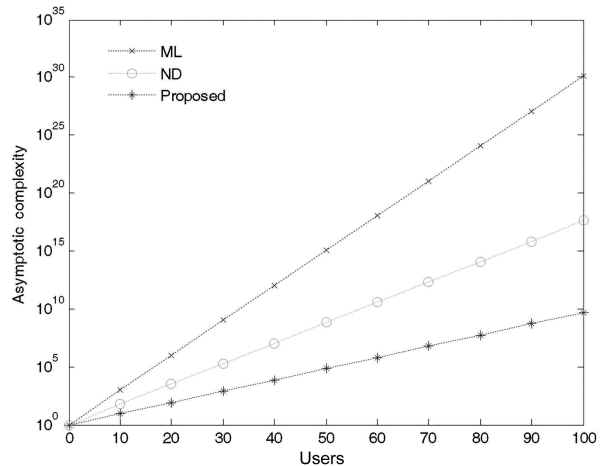


Fig. 5. The computational complexities versus number of users for a mid-capacity network ($0 < K \leq 100$). ML: maximum likelihood, ND: neighbor descent.

A. Analysis of Computational Complexity

According to our numerical results, we successfully reduced the computational complexity at an acceptable BER after considering the DS-CDMA synchronous LTI system. The numerical results show the computational complexities with respect to the number of users as shown in Figs. 4 and 5 for 10 and 100 users, respectively. As the number of users increases in the system, the computational complexity differences among the three approaches become obvious.

Fig. 4 presents the computational complexities for a network that consists of 10 users. As we can see, the proposed algorithm for a small network of 10 users requires fewer computations as compared to the ML and the ND algorithms. In addition, the proposed algorithm greatly reduces the unnecessary computations involved in signal detection by storing the pattern of occurrence of the demodulated bits in the transformation matrix and uses it only on those coordinates or decision regions which are most likely lead to an incorrect decision. The computational complexity for a network that consists of 100 users is shown in Fig. 5.

It should be noted that the computational complexity curve for the proposed algorithm is growing in a linear order rather than in an exponential order. The computational linearity of the proposed PE algorithm comes at the cost of not considering all the decision variables and thus provides much better performance over the ND and the ML algorithms. In other words, this can be considered as an extension of the former results that demonstrate the consistency in the linear growth for the required computations of the proposed algorithm. As we increase the number of users in the system, additional transformation matrixes will be used to determine which coordinate(s) or

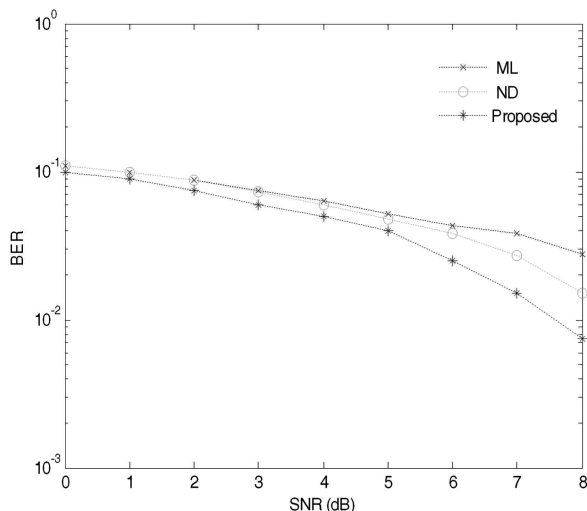


Fig. 6. BER versus SNR ($0 < \text{dB} < 9$) curves for the synchronous direct sequence-code division multiple access (DS-CDMA) based system in an additive white Gaussian noise (AWGN) channel with linear time invariant properties. BER: bit error rate, SNR: signal to noise ratio, ML: maximum likelihood, ND: neighbor descent.

decision region(s) within the constellation diagram is most likely to produce errors.

B. Performance Analysis of BER Performance

Simulation results show that the proposed technique performs better than the ML and the ND algorithms for all values of BER. Figs. 6 and 7 display a plot of three BER versus SNR curves. These curves were plotted in an AWGN channel for a small range of users. It should be noted that the BER performance of the proposed technique is always better than the ML and the ND algorithms as shown in Fig. 6. For the first few values of SNR, the ND algorithm nearly approaches the ML algorithm whereas the proposed technique still maintains a reasonable performance difference. It can be seen in Fig. 6 that the proposed technique achieves less than 10^{-2} BER for SNR = 8 dB which is quite close to the required reasonable BER performance for a voice communication system. For small values of SNR, the BER for these three algorithms is almost equal, but as we increase the value of SNR, typically more than 10 dB, one can clearly observe the difference in the BER performance.

The former result demonstrates a slight improvement over the BER performance shown in Fig. 6 for all SNR values above 9 dB. Even for small values of SNR, the proposed technique demonstrates better performance than the ML and the ND algorithms. As the value of SNR increases, the BER performance of the proposed technique over the ND and the ML algorithms becomes more and more substantial because the probability of having more divergent values of SNR increases. It can also be noticed in Fig. 7 that the proposed algorithm achieves

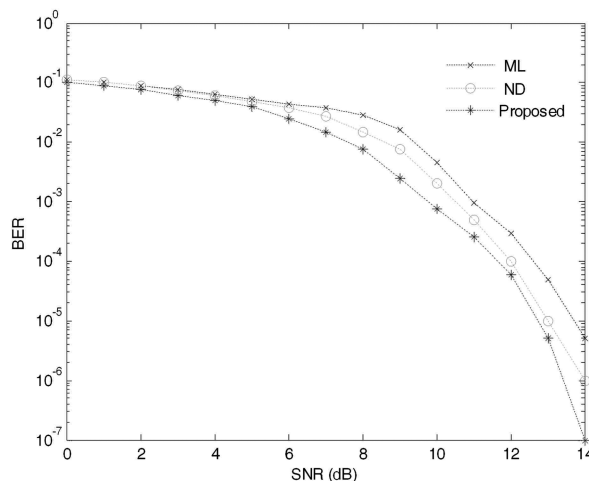


Fig. 7. BER versus SNR ($0 < \text{dB} < 14$) curves for synchronous direct sequence-code division multiple access (DS-CDMA) based system in an additive white Gaussian noise (AWGN) channel with linear time invariant properties. BER: bit error rate, SNR: signal to noise ratio, ML: maximum likelihood, ND: neighbor descent.

less than 10^{-3} BER for SNR = 10 dB, which is more than what we desire for a voice communication system. Furthermore, the proposed technique achieves 10^{-7} BER performance for SNR = 14 dB as shown in Fig. 7. This is more than an acceptable value of BER for data communication such as file transfer protocol (FTP) and is quite close to the desired value of BER (typically 10^{-8} BER is required) for high fidelity digital audio systems.

V. Conclusion

In this paper, we presented a new PE algorithm that utilizes the transformation matrix scheme to improve the performance of next generation wireless receivers. This paper provided an implementation of the proposed PE algorithm with the support of a well driven mathematical model. To prove the low-complexity and the correctness of the PE algorithm, we provided a formal mathematical proof for both the upper and lower bounds of the algorithm. The mathematical proofs for both bounds demonstrated that the complexity of the proposed PE algorithm with any input size is always less than the ML and ND algorithms. The reduction in complexity increases the computing power of a multiuser receiver. Consequently, the increase in computing power would likely result in fast signal detection and error estimation which do not come at the expense of performance. Even though the proposed algorithm requires fewer computations for signal estimation and detection, the practicality of our method and the expected results depend on a realistic system model. For the sake of implementation regarding our proposed algorithm, a more practical asynchronous system would be considered in the future that consists of

non-linear time variant properties of the Gaussian channel with an imperfect power control. Specifically, we will extend the proposed approach for CDMA based non-linear likelihood multiuser detectors for the following conditions: wireless multipath fading channels, known and unknown multiple access interference, broadband data, and power control variations.

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