Bayesian Value of Information Analysis with Linear, Exponential, Power Law Failure Models for Aging Chronic Diseases

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The effective management of uncertainty is one of the most fundamental problems in medical decision making. According to the literatures review, most medical decision models rely on point estimates for input parameters. However, it is natural that they should be interested in the relationship between changes in those values and subsequent changes in model output. Therefore, the purpose of this study is to identify the ranges of numerical values for which each option will be most efficient with respect to the input parameters. The Nonhomogeneous Poisson Process (NHPP) was used for describing the behavior of aging chronic diseases. Three kinds of failure models (linear, exponential, and power law) were considered, and each of these failure models was studied under the assumptions of unknown scale factor and known aging rate, known scale factor and unknown aging rate, and unknown scale factor and unknown aging rate, respectively. In addition, this study illustrated developed method with an analysis of data from a trial of immunotherapy in the treatment of chronic Granulomatous disease. Finally, the proposed design of Bayesian value of information analysis facilitates the effective use of the computing capability of computers and provides a systematic way to integrate the expert's opinions and the sampling information which will furnish decision makers with valuable support for quality medical decision making.

Categories and Subject Descriptors: Software & Applications: Decision Sciences

General Terms: Bayesian value of information, Nonhomogeneous Poisson Process (NHPP), Aging Chronic Diseases, Chronic Granulomatous Disease (CGD)

1. INTRODUCTION

Living things are often plastic during their early development and are moulded by the environment (i.e., individuals vary in survival chances due to differences in genetics, environmental exposures, and gene-environment interactions). Difficulties arise when

specifying causes of death at older ages. Multiple causes of death statistics may more accurately portray mortality when deaths are due to multiple concurrent diseases processes. In general, the human physiological systems can be defined as a collection of more than one organ (i.e., respiratory system, the digestive system), which are composed of more than one part to perform either single or multiple organic function. Since the human physiological systems can therefore experience multiple failures, the successive failure times are of special importance for judging the performance of physiological function over time. The successive times between failures are not necessarily identically distributed [Lawless 1982] and [Manton 1988]. More generally, they can become smaller and smaller (an indication of deterioration), or conversely larger and larger (an indication of survival growth). However, traditional competing risk analysis, which assumed independent risks [Katsahian 2004], does not allow for the analysis of multiple causes of death. In addition, such event durations are the intervals between the diagnosis of a disease and the subsequent mortality due to that disease and the interval between a specific clinical treat and recovery from chronic disease [Hu 1997].

If deterioration is detected, then the decision of when to take the critical intervention treatment, given the costs of treatments and failures, is of fundamental importance. At the time of a decision, the degree of future deterioration, which is likely to be uncertain, is of primary interest for the decision maker (i.e., determining the prevalence of disease, doing a population survey, or measuring the level of a toxin) [Albisser 2002]. Uncertainties about the initial status of human physiological systems, life time, the latent period, medical cost, etc., are also important factors [Sendi 2002]. Bayesian value of information analysis can provide methods to deal with these uncertainties. There are two reasons for Bayesian value of information analysis to be the appropriate approach for this study. First, failure time data are not always rich enough to perform a traditional statistical analysis with significant power (i.e., some human physiological functions may fail only once or twice in past several years). Secondly, many aging chronic diseases problems arise in situations where there is a high level of uncertainty, either because of the influence of uncertain factors which are hard to quantify, or because of the survival prediction problem involves significant extrapolation from any available data. However, gathering additional data will not always be economical. There are two pervasive problems in the analysis of such duration data are the loss to follow-up of the study persons due to withdrawal from the study, and the loss due to premature death from other causes [Chang 2007].

The objective of this study is to determine analytically or numerically the conditions under which collecting additional information will be worthwhile. Further, to identify the ranges of numerical information values for which each option will be most efficient with respect to the input parameters, such as actions of risk management, cost of collecting additional information, uncertainty about the initial failure rate and trend of the human physiological systems. Hence, this study provides a better understanding of the different behaviors associated with each model, and a decision process that can facilitate the development of guidelines for chronic diseases risk management.

Section 2 presents a review of literature relevant to the concept of value of

information. Section 3 and 4 describe some basic concepts about deterioration along with three kinds of failure models (linear, exponential, and power law). The results of Bayesian value of information analysis are also presented. Section 5, presents a sample application of the models developed in section 3 and 4. Finally, Section 6 concludes the discussion of this study.

2. BAYESIAN VALUE OF INFORMATION

Bayesian decision theory and value of information analysis provides an analytical framework that can be used to establish the value of acquiring additional information to inform a decision problem. These methods have firm foundations in statistical decision theory [Raiffa 1959] and [Pratt 1995] and have been successfully used in many areas of research such as related engineering and environmental risk analysis [Thompson 1997]. Recently, these methods have been extended to setting priorities in the evaluation of healthcare technologies [Claxton 2001]. Consider a decision problem that has many alternative actions available to be chosen and many possible states of nature. The decision maker is to choose the action that will result in the minimal expected loss with respect to the entire set of possible states of nature [Feltham 1968]. However, taking action right away is not the only alternative because there is always another option open to decision maker, the option of gathering additional information before making a decision. This action does not actually change the state of nature, but provides a more solid basis for forecasting it [Claxton 2001].

The value of information is a concept used in decision analysis to denote the most a decision maker should be willing to pay to resolve some uncertainty. The concept of value of information is a common topic in decision theory. [Raiffa 1968] described "*...the increase (decrease) in utility (loss) which would result if the decision maker learned that Z=z (in the light of the additional information) and therefore altered his prior choice of an act; and we can then take a weighted average of these utility increases (loss decrease). The increase in utility (decrease in loss) which results or would result from learning that Z=z will be called the value of the information z*". [Flockhart 1993] pointed out that "*the value of information is not intrinsic; rather, it is entirely dependent on the usefulness of the information for decision making*". In the light of significant uncertainty, the option of gathering additional information is likely to be desirable and the value of information is likely to be positive.

When the wrong decision is made, there will be costs in terms of health benefit and resources forgone. In general, the expected cost of uncertainty is determined jointly by the probability that a decision based on existing information. It can be interpreted as the expected value of perfect information (*EVPI*), since perfect information can eliminate the possibility of making the wrong decision. If problem is to maximize gains in health outcome subject to a budget constraint then this is also should be willing to pay for additional evidence to inform this decision in the future, and it places an upper bound on the value of conducting further research [Claxton 1999]. The *EVPI* is simply the difference between the payoff (expected net benefit) with perfect and current information [Fenwick 2000] and [Ades 2002]. Since, the *EVPI* can be worked out directly from the simulated output from our model as it relates to

the individual patient. In a word, the *EVPI* represents the value of completely eliminating uncertainty (i.e., collecting information with perfect accuracy).

3. LIKELIHOOD FUNCTIONS OF AGING CHRONIC DISEASES

In terms of Bayesian decision theory, the payoff is the loss function and the diagnostic of data source is represented by the likelihood function. According to [Chang C. C. 2006, 2007], the human physiological failure process is given by the NHPP. The joint density function of the first N failure times is

$$f_{X_1,X_2,\ldots,X_N,N}(x_1,x_2,\ldots,x_n,n)=[\prod_{i=1}^{n}\lambda(x_i)]\,exp(-\Lambda(x^*)] \tag{1}$$

where x^* is a constant, N is a random variable and $\Lambda(x)=\int_0^x \lambda(u)\,du$ is the mean number of failures by time x in the NHPP. The likelihood functions of $Lik(x_1,x_2,\ldots,x_n,n|\lambda_0,\beta)=\lambda_0^n[\prod_{i=1}^{n}(1+\beta x_i)]\,exp[-\lambda_0(x^*+\beta x^{*2}/2)]$ for linear failure model, $Lik(x_1,x_2,\ldots,x_n,n|\lambda_0,\beta)=\lambda_0^n\beta^n(\prod_{i=1}^{n}x_i)^{\beta-1}\,exp(-\lambda_0 x^{*\beta})$ for exponential and $Lik(x_1,x_2,\ldots,x_n,n|\lambda_0,\beta)=\lambda_0^n exp(\beta\sum_{i=1}^{n}x_i)\,exp[-(\lambda_0/\beta)\,exp(\beta x^*)-1]$ for power law failure model, respectively.

In order to model medical decision making, the crucial decision is whether after some period of time t, the failure rate of the physiological systems will be too high (in which case perform some intervention treatment), or will still be within an acceptable range (in which case under the status quo). Another option is to gather additional information. We also assume that the decision maker is risk neutral, and can therefore make the decision on the basis of expected monetary value. The basic elements of the Bayesian decision analysis are: (1) Parameter space Θ: $\{(\lambda_0,\beta)|\lambda_0>0\}$, where λ_0 is the scale factor and β is the aging rate. Both parameters are uncertain and can be estimated through experts' opinions. (2) Action space A: $\{a_1, a_2\}$, where a_1 is the status quo, and a_2 is undertaking the intervention treatment. (We eventually expand this to consider a third possible action, the collection of additional information). (3) Loss function L: the real function defined on $\Theta\times A$. If we decide to keep the status quo, then the loss is $L(\theta,a_1)$; if we decide to undertaking the intervention treatment, then the loss we face is $L(\theta,a_2)$. (4) Sample space S: the additional information available to be collected. This information could be actual data (e.g., successive failure times), or else information obtained by more detailed analysis of existing data (e.g., more detailed root cause analysis of observed events).

In addition, the cost of collecting this additional information should also be reflected in the Bayesian decision process. The detailed analysis descriptions of each part are:

(1) The Prior Analysis: The available prior knowledge (e.g., expert opinion, past experience, or the status of similar chronic disease) about the parameter space, Θ:$\{(\lambda_0,\beta)|\ \lambda_0>0\}$, can be represented by a joint distribution indicating the relative likelihood of each state of nature. Loss functions appropriate for the status quo (a_1) and undertaking the intervention treatment (a_2) can be derived by taking all cost-related data into account. Once the prior distribution and loss function have been specified, it is simple to perform a prior analysis by simply comparing the expected losses for the options a_1 and a_2. If $\boldsymbol{E}\{L(\theta,a_1)\}\geq\boldsymbol{E}\{L(\theta,a_2)\}$, then option a_2 is optimal, and opposite if $\boldsymbol{E}\{L(\theta,a_1)\}<\boldsymbol{E}\{L(\theta,a_2)\}$, then option a_1

is optimal.

(2) The Value of Perfect Information: It assumed that there is an ideal experiment that will yield perfect information concerning the true state Θ, then we can simply choose the option that minimizes the loss according to the exact state θ; i.e., if $L(\theta,a_1)\geq L(\theta,a_2)$, then option a_2 is optimal, and if $L(\theta,a_1)<L(\theta,a_2)$, then option a_1 is optimal. There are two sets of states of nature can be identified, say Θ_1 and Θ_2, such that we have $\theta\in\Theta_1\Leftrightarrow L(\theta,a_1)<L(\theta,a_2)$ and $\theta\in\Theta_2\Leftrightarrow L(\theta,a_1)\geq L(\theta,a_2)$, where $\Theta_1\cup\Theta_2=\Theta$ and $\Theta_1\cap\Theta_2=\varnothing$. If the state of nature has a continuous prior distribution as given by $f_\Theta(\theta)$, then the $EVPI$ is given by $\boldsymbol{E}\{L(\theta,a_1)\}-\{\int_{\Theta_1}L(\theta,a_1)f_\Theta(\theta)d\theta+\int_{\Theta_2}L(\theta,a_2)f_\Theta(\theta)d\theta\}$, when $\boldsymbol{E}\{L(\theta,a_1)\}<\boldsymbol{E}\{L(\theta,a_2)\}$, and $\boldsymbol{E}\{L(\theta,a_2)\}-\{\int_{\Theta_1}L(\theta,a_1)f_\Theta(\theta)d\theta+\int_{\Theta_2}L(\theta,a_2)f_\Theta(\theta)d\theta\}$, when $\boldsymbol{E}\{L(\theta,a_1)\}\geq\boldsymbol{E}\{L(\theta,a_2)\}$. If the loss function is linear in decision variable (e.g., $L(\theta,a_1)\equiv L(W,a_1)=K_1W+k_1$ and $L(\theta,a_2)\equiv L(W,a_2)=K_2W+k_2$), and if the decision variable $W\equiv W(\theta)$ is monotonically increasing in θ, then the $EVPI$ can be simplified to $|K_2-K_1|\int_{w_C}^{\infty}(w-w_C)f_W(w)dw$, when $\boldsymbol{E}\{W\}\leq w_C$, and $|K_2-K_1|\int_{-\infty}^{w_C}(w_C-w)f_W(w)dw$, when $\boldsymbol{E}\{W\}>w_C$, where $w_C=(k_1-k_2)/(K_2-K_1)$.

(3) The Pre-posterior Analysis: When the expected losses associated with options a_1 and a_2 are fairly close, decision maker might not feel very confident about a decision based solely on a prior analysis. If $EVPI$ can be calculated and is greater than 0, then gathering additional information might be desirable. However, decision maker has to investigate the possible outcomes and costs of each candidate sampling plan, to determine whether collecting additional information is worthwhile and also which sampling plan is the best in terms of cost-effectiveness. The expected value of sample information ($EVSI$) can be calculated according to

$$EVSI=\min_{j=1,2}\boldsymbol{E}\{L(\theta,a_j)\}-\min_i\{\boldsymbol{E}_S\{\min_{j=1,2}E\{L(\theta,a_j)|S^{(i)}\}\}+CI(S^{(i)})\} \quad (2)$$

where $S^{(i)}$ is the ith sampling plan under consideration, and $CI(S^{(i)})$ is the cost of the ith sampling plan. If $EVSI\leq 0$, then it is not worthwhile to collect additional information. Conversely, if $EVSI>0$, then we can start collecting data and prepare for a posterior analysis. (4) The Posterior Analysis: Once the optimal sampling plan, say $S^{(k)}$, and the observed data $S^{(k)}=s^{(k)}$ can then be used to perform a posterior analysis. The decision should then be made in accordance with the strategy that if $\boldsymbol{E}\{L(\theta,a_1)|S^{(i)}=s^{(i)}\}\geq\boldsymbol{E}\{L(\theta,a_2)|S^{(k)}=s^{(k)}\}$, then option a_2 is optimal, and if $\boldsymbol{E}\{L(\theta,a_1)|S^{(i)}=s^{(i)}\}<\boldsymbol{E}\{L(\theta,a_2)|S^{(k)}=s^{(k)}\}$, then option a_1 is optimal. By exploring the relationships among the optimal decision and the extent of uncertainty about deteriorating trends, the conditions under which gathering additional information is worthwhile can be determined, and more generally in developing guidelines for the use of isolating trends in data in risk management.

The following terminology will be used throughout this paper:

C_A: the cost of a failure if it occurs.

C_R: the cost of the undertaking intervention treatment.

C_I: the cost of collecting additional information.

ρ : the reduction in failure rate that would result from the proposed perform some intervention treatment action ($0<\rho<1$).

M: the expected number of failures during the time period $[t,T]$ under the status

quo.

The decision variable we are dealing with is then the expected number of failures during the time period $[t,T]$,

$$M \equiv M(T,t,\lambda_0,\beta) = \int_t^T \lambda(s)\,ds \tag{3}$$

Note that the expected number of failures M is a random variable and it is a function of the two uncertain parameters λ_0 and β. Suppose that undertaking the intervention treatment action will reduce the failure intensity by a fraction ρ, where $0<\rho<1$, and it is given by $\int_t^T \lambda(s)(1-\rho)\,ds = (1-\rho)\ M$. On the basis of the assumptions given above, decision maker therefore has a two-action problem with a linear loss function, where the loss for taking action a_1 (i.e., continuing with the status quo) is $C_A M$ and the loss for taking action a_2 (i.e., undertaking the intervention treatment) is $C_A(1-\rho)M+C_R$. The expected loss for the status quo is simply $C_A E\{M\}$, and the expected loss for undertaking some intervention treatment is $C_A(1-\rho)\boldsymbol{E}\{M\}+C_R$. Finally, Bayesian decision theory and an analysis of the value of information can be used to decide whether the evidence in an economic study is sufficient substantiation.

4. BAYESIAN VALUE OF INFORMATION ANALYSIS

This section discusses the processes of prior and posterior analysis for each of three failure models (linear, power law, and exponential) was studied under the assumptions of unknown scale factor and known aging rate, known scale factor and unknown aging rate, and unknown scale factor and unknown aging rate, respectively.

4.1 The Case of Unknown λ_0 and Known β

This assumption can make the decision analysis more tangible, and therefore help us to study the decision behavior by using sensitivity analysis to vary β, particular when sample information is sparse. From equation (3), and since $M \equiv M(T,t,\lambda_0,\beta) = \lambda_0[H(\beta;T)-H(\beta;t)] = \lambda_0 H$, the prior distribution of M can be easily transformation as $f_M(m) = f_{\lambda_0}\left(\frac{m}{H}\right)/H$. Therefore, the *EVPI* can be simplified with respect to the scale factor λ_0, and is given by

$$\begin{cases} C_A \rho H \int_{\tau_C}^{\infty} (\lambda_0 - \tau_C) f_{\lambda_0}(\lambda_0)\,d\lambda_0 & \text{if } \boldsymbol{E}\{\lambda_0\} \le \tau_C, \text{ and} \\ C_A \rho H \int_0^{\tau_C} (\tau_C - \lambda_0) f_{\lambda_0}(\lambda_0)\,d\lambda_0 & \text{if } \boldsymbol{E}\{\lambda_0\} > \tau_C, \end{cases} \tag{4}$$

where $\tau_C = C_R/\{C_A \rho H\}$ is the cutoff value of $E\{\lambda_0\}$ for undertaking the intervention treatment. If λ_0 is distributed *Uniform(a,b)*, then the *EVPI* is given by $C_A \rho H(\tau_C - a)^2/2d$, when $\boldsymbol{E}\{\lambda_0\} > \tau_C$, and the *EVPI* is given by $C_A \rho H(b-\tau_C)^2/2d$, when $\boldsymbol{E}\{\lambda_0\} \le \tau_C$, where $d=b-a$ represents the uncertainty about λ_0. If the cost of collecting perfect information about λ_0 is within the range $0 < C_I < C_A \rho d H/8$, then collecting information will be desirable when $\boldsymbol{E}\{\lambda_0\} \subseteq \left[\tau_C \pm \left(\frac{d}{2} - \sqrt{\frac{2dC_I}{C_A \rho H}}\right)\right]$. Note that the width of this range is given by $w_I = d - 2\sqrt{\frac{2dC_I}{C_A \rho H}}$, this width will increase as d increases as long as $w_I > 0$; if $w_I \le 0$, then collecting additional information is not worthwhile. Thus, it is more

desirable to collect information when the uncertainty about λ_0 is larger. Also, w_I increases when the failure cost increases, and decreases when the cost of collecting information increases. However, the cost of undertaking the intervention treatment will has no effect on w_I.

(1) The linear failure model: In this model, H is given by $T-t+(1/2)\beta(T^2-t^2)$. Figure 1 shows the *EVPI* about λ_0 for this model as a function of the prior mean $E\{\lambda_0\}$ and the prior standard deviation $SD\{\lambda_0\}$ when λ_0 has a uniform prior distribution. There are three different values for β were used in Figure 1, where $\beta_1<\beta_2<\beta_3$. In addition, Figure 2 shows the results of sensitivity analysis about β. If the prior mean of λ_0 is λ_0^*, then undertaking the intervention treatment should be undertaken when $\beta>\beta_U$, the status quo should be maintained when $\beta\leq\beta_L$, and additional information about λ_0 should be collected before making the decision when $\beta_L<\beta\leq\beta_U$. The time horizon under consideration and the time at which the decision is being made (i.e., T and t) can also affect w_I. In other words, w_I is increasing in T when $\beta>0$ or when $\beta<0$ and $T>-1/\beta$, and is decreasing in T when $\beta<0$ and $T<-1/\beta$.

(2) The power law failure model: In this model, H is given by for the power law model and know from equation (3) that w_I is increasing in β, if $T>1$. Conversely, when $T<1$, w_I is increasing in β when $\beta<ln[ln(t)-ln(T)]/ln(T/t)$, and decreasing

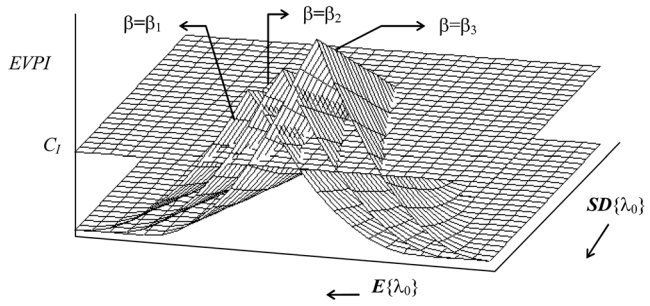


Figure 1. *EVPI* about λ_0 for the Linear Failure Model.

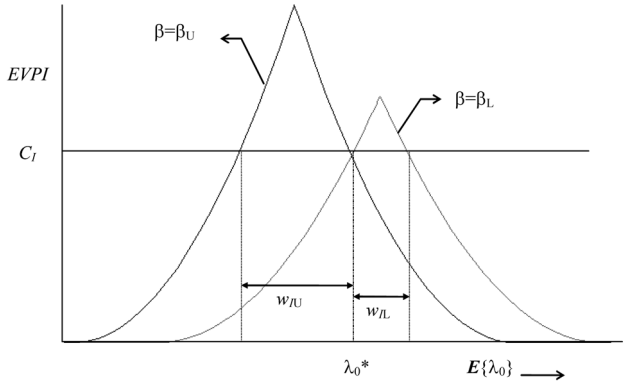


Figure 2. Sensitivity analyze about β for the Linear Failure Model.

when $\beta > ln[ln(t)-ln(T)]/ln(T/t)$. Figure 3 shows the *EVPI* about λ_0 for this model as a function of the prior mean $E\{\lambda_0\}$ and the prior standard deviation $SD\{\lambda_0\}$ when λ_0 has a uniform prior distribution with three different values of β, where $\beta_1<\beta_2<\beta_3$. The time horizon under consideration and the time at which the decision is being made (i.e., T and t) can also affect w_I. Since H is increasing in T and decreasing in t and w_I is increasing in T and decreasing in t, too.

(3) The exponential failure model: In this model, H is given by $[exp(\beta T) - exp(\beta t)]/\beta$ and know from equation (3) that w_I is increasing in β. Figure 4 shows the *EVPI* about λ_0 for this model as a function of the prior mean $E\{\lambda_0\}$ and the prior standard deviation $SD\{\lambda_0\}$, when λ_0 has a gamma prior distribution with three different values of β, where $\beta_1 < \beta_2 < \beta_3$. The time horizon under consideration and the time at which the decision is being made (i.e., T and t) can also affect w_I. Since the parameter H is increasing in T and decreasing in t and w_I is increasing in T and decreasing in t, too.

Nevertheless, it is difficult to determine the *EVPI* in closed form for this case, except when λ_0 has a uniform distribution. It cannot get strictly analytical results, since the results depend on functions such as the incomplete gamma function or the cumulative normal distribution. The functional forms for H corresponding to the various failure models (i.e., the linear, exponential, and power law failure models) can also be substituted into the above expressions to evaluate the *EVPI* with respect

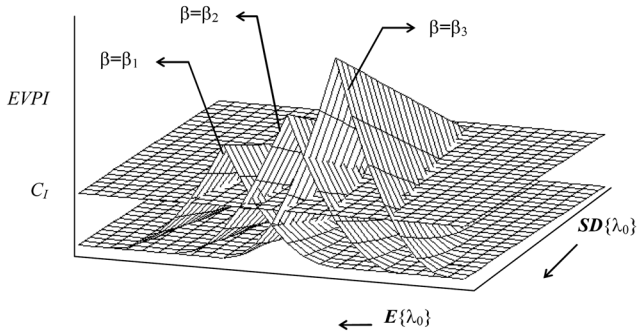


Figure 3. *EVPI* about λ_0 for the Power Law Failure Model.

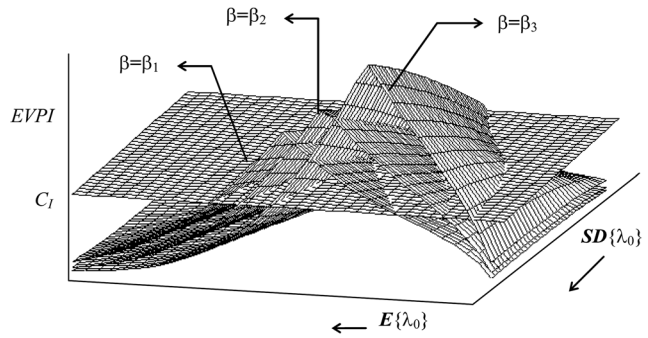


Figure 4. *EVPI* about λ_0 for the Exponential Failure Model.

to different failure models. Unfortunately, when λ_0 has a gamma, lognormal, or Weibull distribution, the range of values of the prior mean $E\{\lambda_0\}$ within which collecting information is desirable cannot be determined in closed form. However, this problem can be determined by numerical integration.

4.2 The Case of Known λ_0 and Unknown β

In the case of unknown β, it is difficult to develop decision processes based on the NHPP model even when λ_0 is known. However, the assumption that λ_0 is known can help decision maker to understand the decision behavior by using sensitivity analysis to vary λ_0. Since $M \equiv M(T,t,\lambda_0,\beta)=\lambda_0[H(\beta;T)-H(\beta;t)]=\lambda_0 \mathsf{H}$, the *EVPI* can be simplified with respect to H and is given by

$$\begin{cases} C_A \rho \lambda_0 \int_{\tau_C}^{\infty} (\mathsf{h}-\tau_C) f_{\mathsf{H}}(\mathsf{h})\, dh & \text{if } E\{\mathsf{H}\} \le \tau_C \\ C_A \rho \lambda_0 \int_{\infty}^{\tau_C} (\tau_C - \mathsf{h}) f_{\mathsf{H}}(\mathsf{h})\, dh & \text{if } E\{\mathsf{H}\} > \tau_C \end{cases} \tag{5}$$

where $\tau_C = C_R / \{C_A \rho \lambda_0\}$ is the cutoff value of $E\{\mathsf{H}\}$ at which undertaking some intervention treatment action should be adopted.

(1) The linear failure model: In this model, H is given by $\mathsf{H}(\beta)=H(\beta;T)-H(\beta;t)$. The prior density function of H can be derived by transformation of variables as follows,

$$f_{\mathsf{H}}(\mathsf{h}) = f_{\beta}(\mathsf{H}^{-1}(\mathsf{h})) \left| \frac{d\mathsf{H}^{-1}(\mathsf{h})}{d\mathsf{h}} \right| \tag{6}$$

Once the parameter H is monotonic with β, the prior density function of H for the linear failure model is of the form $f_{\mathsf{H}}(\mathsf{h})=2f_{\beta}(2(\mathsf{h}-T+t)/(T^2-t^2))/(T^2-t^2)$, and β is distributed *Uniform*(a,b), then the *EVPI* is

$$\frac{C_A \rho \lambda_0 [\tau_C - T + t - a(T^2-t^2)/2]^2}{d(T^2-t^2)} \tag{7}$$

when $\boldsymbol{E}\{\mathsf{H}\} > \tau_C$, and

$$\frac{C_A \rho \lambda_0 [T - t + b(T^2-t^2)/2 - \tau_C]^2}{d(T^2-t^2)} \tag{8}$$

when $\boldsymbol{E}\{\mathsf{H}\} \le \tau_C$, where $d=b-a$ represents the uncertainty about β. If C_I satisfies $0 < C_I < \frac{C_A \rho \lambda_0 d(T^2-t^2)}{16}$, then collecting information about β will be desirable, when $\boldsymbol{E}\{\mathsf{H}\} \subseteq \left[\tau_C \pm \left(\frac{d(T^2-t^2)}{4} - \sqrt{\frac{C_I d(T^2-t^2)}{C_A \rho \lambda_0}} \right) \right]$, the width of this range is given by

$$w_I = \frac{d(T^2-t^2)}{2} - 2\sqrt{\frac{C_I d(T^2-t^2)}{C_A \rho \lambda_0}}. \tag{9}$$

In the equation (9), this width will increase when either d or T^2-t^2 as long as $w_I>0$; if $w_I \le 0$, then collecting additional information is not worthwhile. Thus, it is more desirable to collect data when the uncertainty about β is larger, and when the time horizon under consideration is longer. Also, w_I increases when the cost of accident

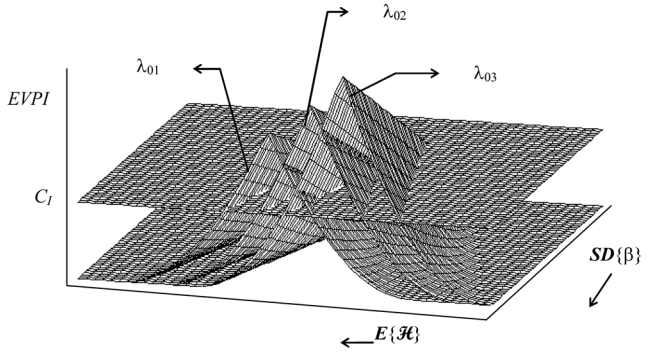


Figure 5. *EVPI* about β for the Linear Failure Model.

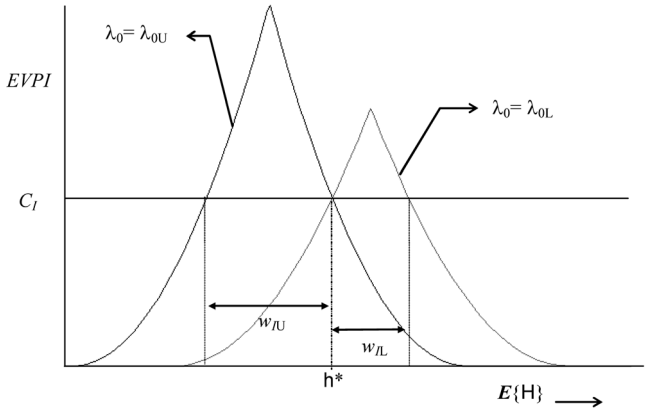


Figure 6. Sensitivity analyze about λ_0 for the Linear Failure Model.

increases, and decreases when the cost of collecting information increases. However, the cost of undertaking the intervention treatment has no effect on w_I. Figure 5 shows the *EVPI* about β for the linear failure model as a function of the prior mean $E\{\mathsf{H}\}$ and the prior standard deviation $SD\{\beta\}$ when β has a uniform prior distribution with three different values of λ_0, where $\lambda_{01}<\lambda_{02}<\lambda_{03}$. In addition, Figure 6 shows the results of sensitivity analysis about λ_0 for the linear failure model. If the prior mean of H is h^*, then undertaking the intervention treatment should be undertaken when $\lambda_0>\lambda_{0U}$, the status quo should be maintained when $\lambda_0\leq\lambda_{0L}$, and information about β should be collected before making the decision when $\lambda_{0L}<\lambda_0\leq\lambda_{0U}$.

Nevertheless, it is difficult to determine the *EVPI* in closed form for linear failure model, except when β has a uniform distribution. For most other distributions it cannot get strictly analytical results, since the results depend on functions such as the incomplete gamma function or the cumulative normal distribution.

(2) The power law failure model: In this model, H is given by $\mathsf{H}(\beta)=H(\beta;T)-H(\beta;t)$. The inverse function $\mathsf{H}^{-1}(\mathsf{h})$ cannot be found analytically for this model. Therefore, no analytical functional form is available for *EVPI* with respect to the aging rate β, and numerical computation is needed to determine the *EVPI* in this

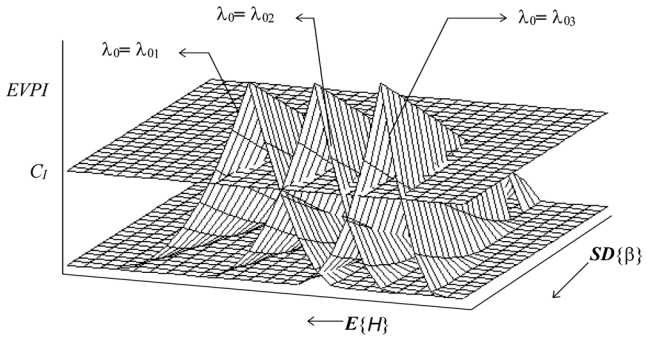


Figure 7. *EVPI* about β for the Power Law Failure Model.

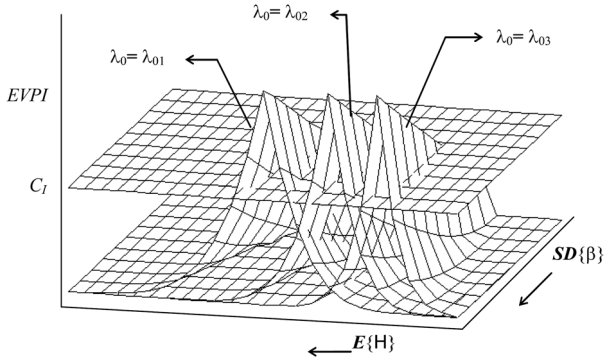


Figure 8. *EVPI* about β for the Exponential Failure Model.

cases. Figure 7 shows the *EVPI* about β for the power law failure model as a function of the prior mean $E\{H\}$ and the prior standard deviation $SD\{\beta\}$ when β has a uniform prior distribution with three different values of λ_0, where $\lambda_{01}<\lambda_{02}<\lambda_{03}$.

(3) The exponential failure model: In this model, H is given by $\mathsf{H}(\beta)=H(\beta;T)-H(\beta;t)$. The inverse function $\mathsf{H}^{-1}(\mathsf{h})$ cannot be found analytically for this failure model. Therefore, no analytical functional form is available for *EVPI* with respect to the aging rate β, and numerical computation is needed to determine the *EVPI* in this cases. Figure 8 shows the *EVPI* about β for the exponential failure model as a function of the prior mean $E\{\mathsf{H}\}$ and the prior standard deviation $SD\{\beta\}$ when β has a gamma prior distribution with three different values of λ_0, where $\lambda_{01}<\lambda_{02}<\lambda_{03}$.

4.3 The Case of Unknown λ_0 and Unknown β

In real world situations, it will generally be desirable to incorporate uncertainty into both λ_0 and β. The assumption that information can be obtained about either λ_0 or β individually is not necessarily realistic, since the information about which decision maker collect is likely to come from actual anamnesis. Thus, the process of gathering information will often provide us with joint knowledge about both λ_0 and β. Since scarcity of data can be partially compensated by careful selection of an informative

prior. In general, as a simplistic assumption, one can assume that λ_0 and β are independent of each other. With this assumption, the joint distribution of λ_0 and β is just the product of the individual distributions of λ_0 and β. Since the *EVPI* is given by

$$\begin{cases} C_A\rho \int_{M_C}^{\infty} (m-M_C) f_M(m)\,dm & \boldsymbol{E}\{M\} \le M_C, \text{ and} \\ C_A\rho \int_{\infty}^{M_C} (M_C-m) f_M(m)\,dm & \boldsymbol{E}\{M\} > M_C, \end{cases} \tag{10}$$

where M is the expected number of failures during the time period $[t,T]$ under the status quo, and $M_C=C_R/(C_A\rho)$ is the cutoff value of $E\{M\}$ for undertaking the intervention treatment. Since M is monotonic in λ_0, the density function of M can be derived by bivariate transformation of variables as follows:

$$f_M(m)= \int_{\beta} f_{\lambda_0,\beta}(m/\mathsf{H},\, \beta)/\mathsf{H}\, d\beta\,, \tag{11}$$

where $\mathsf{H} = \mathsf{H}(\beta)=H(\beta;T)\text{-}H(\beta;t)$ is the cumulative aging-time function over the period $[t,T]$. Since the prior distributions of λ_0 and β are assumed to be independent, the prior density function of M is given by

$$f_M(m)= \int_{\beta} f_{\lambda_0}(m/\mathsf{H}) f_{\beta}(\beta)/\mathsf{H}\, d\beta\,. \tag{12}$$

The functional forms for H corresponding to the various failure models can be substituted into the above expressions, along with prior distributions for both λ_0 and β. The *EVPI* for each failure model can then be evaluated. For most combinations of distributions for λ_0 and β, closed form expressions for the *EVPI* are not available. However, numerical integration can be used to obtain numerical values for use in decision analysis. Figure 9 shows the *EVPI* for the linear failure model as a function of the prior mean $E\{M\}$ and the prior standard deviation $SD\{\lambda_0\}$ when λ_0 has a gamma prior distribution and b has a uniform prior distribution.

5. COMPUTER SIMULATION

This section will serves as a case study to illustrate the use of the models developed in the previous section and discusses the processes of prior and posterior analysis for

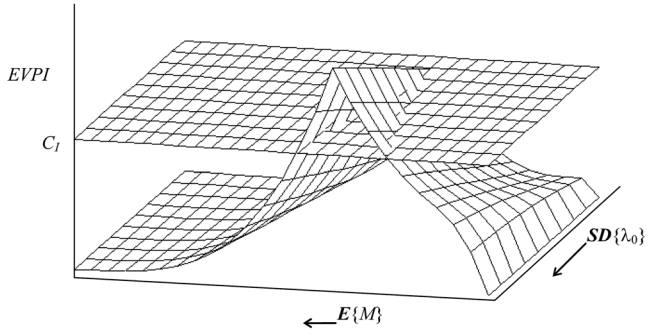


Figure 9. *EVPI* for the Linear Failure Model when λ_0 and β are independent.

each of three failure models was studied under the assumptions of unknown scale factor and known aging rate, known scale factor and unknown aging rate, and unknown scale factor and unknown aging rate, respectively. In addition, the range within which collecting information is desirable, the expected value of perfect information, the optimal sampling time, and the prior and posterior decisions are investigated for each model in each case to assess the effects of prior knowledge about λ_0 and β.

Real failure data from a trial of immunotherapy for the treatment of Chronic Granulomatous Disease (CGD) are studied [International Chronic Granulomatous Disease Cooperative Study Group 1991]. CGD is an inherited disease caused by defects in superoxide-generating nicotinamideadenine dinucleotide phosphate (NADPH) oxidase of phagocytes. Impairment of oxygen-dependent intracellular killing mechanisms results in severe bacterial or fungal infections with catalase-producing Staphylococcus aureus, Burkholderia cepacia, or Aspergillus spp. Antimicrobial prophylaxis is efficient in reducing the incidence of severe bacterial infections. However, fungal infections remained the main cause of mortality in CGD. In one study the use of prophylactic itraconazole reduced the incidence of fungal infections but the effectiveness of long-term prophylaxis remains to be evaluated. Patients with CGD benefit from recombinant interferon-γ(rIFN-γ) prophylaxis. In developed countries, survival of CGD patients has been improved with more patients surviving into third decade of life. However, premature mortality is still the hallmark of CGD. In developing countries, both delay in diagnosis of CGD and poor compliance with long-term antimicrobial prophylaxis are responsible for high morbidity and premature mortality.

The initial date of the studied subject was 1-May-1973, and the observation period was from 24-August-1988, to 1-Sep-1989. The failure dates for the subject during the observation period were: 26-Sep-88, 26-Oct-88, 25-Nov-88, 25-Dec-88, 24-Jan-89, 23-Feb-89, 25-Mar-89, 24-Apr-89, 5-May-89, 24-May-89, 23-Jun-89, 23-Jul-89, 15- Aug-89, 22-Aug-89. Throughout the time unit is taken to be years, time horizon under consideration is 20 years (i.e., T=20), and the time at the decision is being made is 14.417 years after the Birth date (i.e., t=14.417). The cost of a possible undertaking some intervention treatment action can be evaluated by considering the availability cost, the required magnitude of undertaking some intervention treatment, and the desired success probability of the action. It assumed that the cost of a failure if it occurs is \$100,000, the cost of the undertaking some intervention treatment action is \$75,000 (i.e., C_A=100,000 and C_R=75,000), and the reduction in failure intensity that would result from the undertaking some intervention treatment action is 0.1 (i.e., ρ=0.1). Note that the number of failures during the remaining lifetime of CGD can be large, and therefore, compared with the cost of undertaking some intervention treatment action, maintaining the status quo can also be expensive.

5.1 The Case of Unknown λ_0 and Known β

In the case of unknown λ_0 and known β, an appropriate value of β may be obtained from previous clinical experience or expert opinion. It assumed that λ_0 has the same

gamma distribution for the three failure models, with expected value 0.1 and standard deviation 0.08 (i.e., $E\{\lambda_0\}$=0.1 and $SD\{\lambda_0\}$=0.08). However, β can take on different values for the three failure models, since each model has its own way of describing the aging process. It assume that β=0.6 for the linear failure model, β=1.65 for the power law failure model, and β=0.16 for the exponential failure model. These parameters values were chosen to ensure that the three intensity functions were reasonably close to each other during the observation period. In other words, the three failure models were chosen to give similar results within the observation period, and also to be reasonably consistent with the observed data. Figure 10 shows the assumed mean failure intensity functions for the three failure models.

Note that β is measured in units of 1/year for the linear and exponential failure models, and is unitless for the power law failure model. Since there have C_A=100,000, C_R=75,000, ρ=0.1, T=20, and t=14.417, and the distribution of λ_0. Further, it can determine the *EVPI* for each failure model using equation. The *EVPI* can therefore be compared with the cost of collecting additional information to investigate whether collecting such information would be desirable. It also assumed that the cost of collecting perfect information is $10,000 (i.e., C_I=10,000).

(1) Linear Failure Model: The failure process is modeled by the linear failure model with β=0.6, then the *EVPI* is $14,995.45. Since the *EVPI* is nearly 50% greater than the assumed cost of perfect information, gathering such information would be desirable. Moreover, if let holding the standard deviation constant at $SD\{\lambda_0\}$ =0.08, the range of prior expectations $E\{\lambda_0\}$ within which collecting information is desirable is given by [0.0688, 0.1500], so collecting additional information would still be desirable even if $E\{\lambda_0\}$ were nearly one-third lower than its assumed value of 0.1. Sensitivity analysis can also be used to give the range values of β within which collecting information is desirable. The results of β is less than 0.5085, the status quo should be maintained; if β is greater than 1.5437, the intervention treatment should be undertaken; and if β is within the range [0.5085, 1.5437], then collecting additional information is desirable. Thus, a large increase from the assumed value of β=0.6 would be needed to justify an immediate

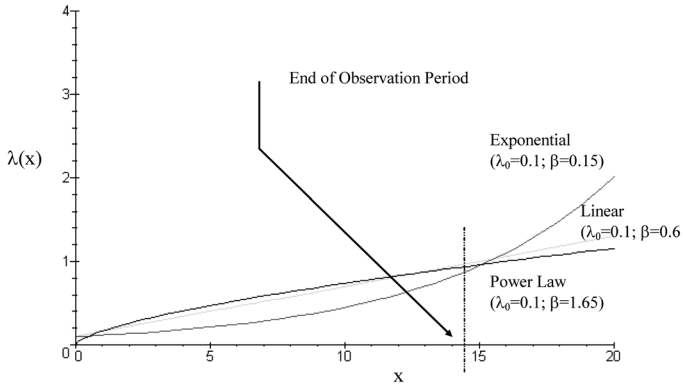


Figure 10. The mean of Failure Intensity Functions for Linear, Power Law, and Exponential Failure Models.

undertaking some intervention treatment action, while if β were only 10% lower than its assumed value, the status quo would be clearly acceptable.

(2) Power Law Failure Model: The failure process is modeled by the power law failure model with β=1.65, then the *EVPI* is \$12,215.69. Since the *EVPI* is more than 20% greater than the assumed cost of perfect information, gathering such information would be desirable. Moreover, if let holding the standard deviation constant at $SD\{\lambda_0\}$=0.08, the range of prior expectations $E\{\lambda_0\}$ within which collecting information is desirable is given by [0.0848, 0.1568], so collecting additional information would still be desirable even if $E\{\lambda_0\}$ were more than 15% lower than its assumed value of 0.1. Sensitivity analysis can also be used to give the range of values of β within which collecting information is desirable. The results are that if β is less than 1.6292, the status quo should be maintained; if β is greater than 1.9339, the undertaking intervention treatment action should be undertaken; and if β is within the range [1.6292, 1.9339], then collecting additional information is desirable. Thus, a large increase from the assumed value of β=1.65 would be needed to justify an immediate undertaking some intervention treatment, while if β were only about 1% lower than its assumed value, the status quo would be clearly acceptable.

(3) Exponential Failure Model: The failure process is modeled by the exponential failure model with β=0.16, then the *EVPI* is \$18,654.01. Since the *EVPI* is more than 85% greater than the assumed cost of perfect information, gathering such information would be desirable. Moreover, if let holding the standard deviation constant at $SD\{\lambda_0\}$=0.08, the range of prior expectations $E\{\lambda_0\}$ within which collecting information is desirable is given by [0.0212, 0.1245], so collecting additional information would still be desirable even if $E\{\lambda_0\}$ were more than 75% lower than its assumed value of 0.1. Sensitivity analysis can also be used to give the range of values of β within which collecting information is desirable. The results are that if β is less than 0.1310, the status quo should be maintained; if β is greater than 0.1900, the undertaking intervention treatment action should be undertaken; and if β is within the range [0.1310, 0.1900], then collecting additional information is desirable. Thus, a nearly 19% increase from the assumed value of β=0.16 would be needed to justify an immediate undertaking some intervention treatment action, while if β were about 18% lower than its assumed value, the status quo would be clearly acceptable.

5.2 The Case of Known λ_0 and Unknown β

In the case of known λ_0 and unknown β, an appropriate value of λ_0 may be obtained from previous clinical experience or expert opinion. It assumed that aging (rather than survival growth) is sure to occur for CGD. Furthermore, it assume that β is *Uniform*[0, 1.2] for the linear failure model, *Uniform*[1, 2.3] for the power law failure model, and *Uniform*[0, 0.32] for the exponential failure model. These assumptions are made to ensure that the mean values of β for these models are equal to the values of β used in the previous section, and that the lower bounds are the minimal values consistent with aging rather than survival growth. It also assumed that λ_0=0.1 for all

three failure models. Since we have C_A=100,000, C_R=75,000, ρ=0.1, T=20, and t=14.417, and the distribution of β, it also assumed that the cost of collecting perfect information is $10,000. The *EVPI* can therefore be compared with the cost of collecting additional information to investigate whether collecting such information would be desirable. The three failure models all have the same cutoff value of $E\{M\}$ for taking the undertaking some intervention treatment action, which is given by $M_C=C_R/(C_A\rho)=7.5$.

Table I summarizes the results of the data analyses for the case of known λ_0 and unknown β. The observed data support the undertaking intervention treatment action for all failure models, whereas the priors support the adoption of the undertaking intervention treatment action only for the power law and exponential failure models. This can be explained by the fact that the observed data indicate greater aging than was assumed by the prior distributions, since the posterior means of β are greater than the prior means of β for all failure models. Furthermore, since the functional form of M is more sensitive to the value of β for the power law and exponential failure models than for the linear failure model. Hence, the uncertainty about β makes the undertaking intervention treatment action worthwhile for the other two models but not for the linear failure model. This also explains why the optimal prior decisions are different for the power law and the linear failure models even though their mean failure intensity functions are closed.

5.3 The Case of Unknown λ_0 and Unknown β

In this section, it considered the case in which both λ_0 and β are unknown. In the

Table I. Decision table for the case of known λ_0 and unknown β.

	Linear Failure Model	Power Law Failure Model	Exponential Failure Model
Prior E{M}	6.3228	11.7914	27.2395
Range of E{M} for Collecting Information	6.5375~8.4625	4.1569~35.8163	3.6850~82.1073
Assumed Value of λ_0	0.1	0.1	0.1
Range of λ_0 for Collecting Information	0.1022~0.1822	0.0358~0.3304	0.0124~0.2924
EVPI	9,125.86	26,228.35	22,373.73
Prior E{β}	0.6	1.65	0.16
Posterior E′{β}	0.9148	1.7943	0.2164
Cutoff Value of E{M} for Intervention Treatment	7.5	7.5	7.5
Prior Decision	Status Quo	Intervention Treatment	Intervention Treatment
Posterior E′{M}	9.3474	10.2215	27.8104
Posterior Decision	Intervention Treatment	Intervention Treatment	Intervention Treatment

case where λ_0 and β are independent, it assumed that λ_0 has the same gamma distribution for all three failure models, with expected value 0.1 and standard deviation 0.08, and that β is *Uniform*[0, 1.2] for the linear failure model, *Uniform*[1, 2.3] for the power law failure model, and *Uniform*[0, 0.32] for the exponential failure model. Therefore, λ_0 has the same distribution, and β has the same distribution for each failure model.

Therefore, it have C_A=100,000, C_R=75,000, ρ=0.1, T=20, and t=14.417, and the distributions of λ_0 and β, it can evaluate the *EVPI* and compared with the cost of collecting additional information to investigate whether collecting such information would be desirable. It also assumed that the cost of collecting perfect information about both λ_0 and β is \$15,000. This is greater than the cost of collecting perfect information about either λ_0 or β individually, but smaller than the sum of these costs, since there may be some repetitive costs that can be eliminated when collecting additional information about both λ_0 and β. Three failure models have the same cutoff value of $E\{M\}$ for taking the undertaking intervention treatment action, which is given by $M_C=C_R/(C_A\rho)$=7.2. The optimal sampling time and the expected net gain of sample information are not available, because the joint probability density function of the sample data is too complicated. Nevertheless, since the failure data are available, we use the entire failure data for the posterior analysis. Prior and posterior analyses are performed by comparing the prior and posterior mean values of M with the cutoff value M_C. Table II summarizes the results of the decision analyses under the assumption of independence between λ_0 and β.

Table II. Decision table for the case of unknown λ_0 and unknown β.

	Linear Failure Model	Power Law Failure Model	Exponential Failure Model
Prior E{M}	6.1428	11.0327	26.4416
Range of E{M} for Collecting Information	5.327~10.152	2.538~27.968	3.622~55.860
EVPI	19,046.56	32,231.42	28,224.74
Prior E{λ_0}	0.1	0.1	0.1
Posterior E′{λ_0}	0.1694	0.1422	0.17
Prior E{β}	0.5	1.57	0.16
Posterior E′{β}	0.7849	1.8488	0.1809
Cutoff Value of E{M} for Intervention Treatment	7.2	7.2	7.2
Prior Decision	Status Quo	Intervention Treatment	Intervention Treatment
Posterior E′{M}	9.9132	10.2495	23.4724
Posterior Decision	Intervention Treatment	Intervention Treatment	Intervention Treatment

5.4 Discussions

In the case of unknown λ_0 and known β, for the base case the width of values of λ_0 within which collecting additional information is desirable is larger for the exponential failure model than for either the linear failure model or the power law failure model. Similarly, the *EVPI* is larger for the exponential failure model than for the other failure models for the base case. These results suggest that the possibility of rapid aging with the exponential failure model may make reduction of uncertainty more important, as one might expect (although it would not have been entirely clear a priori whether decision maker should expect the possibility of rapid aging to favor data collection or the immediate adoption of the intervention treatment).

In the case of known λ_0 and unknown β, the width of the range of values of $E\{M\}$ within which collecting additional information is desirable is much larger for both the power law failure model and the exponential failure model than for the linear failure model. This is because the functional form of M is more sensitive to the value of β for the power law and exponential failure models than for the linear failure model. The range of values of λ_0 within which collecting additional information is desirable is also larger for the power law and exponential failure models than for the linear failure model. Finally, the *EVPI* is larger for both the power law and exponential failure models than for the linear failure model. These results again show the importance of reducing uncertainty when rapid aging is possible as is intuitively reasonable. Similar results are also found in the case of unknown λ_0 and unknown β.

Overall, the case of unknown λ_0 and unknown β represents greater uncertainty than the other two cases, since the *EVPI* for the case of unknown λ_0 and unknown β is larger than for the other two cases. Thus, even with the linear failure model (where the prior decision is always to maintain the status quo), the optimal posterior decision is to undertake the intervention treatment.

6. CONCLUSIONS

The expected costs of uncertainty are determined by the probability that a treatment decision based on existing information will be wrong and by the consequences if the wrong decision is made. This paper develops Bayesian value of information analysis procedures for risk management of aging chronic diseases, without arbitrarily assuming that human physiological systems are perfectly renewed by each repair. The proposed decision models can provide decision support techniques not only for taking action in the light of all available relevant information, but also for minimizing expected loss. In this study, three parametric failure models (the linear, power law, and exponential failure models) are studied to give a better understanding of the differing behavior associated with each model. The power law and exponential failure models appear to be more sensitive to the aging rate than the linear failure model. In addition, the exponential failure model may be less realistic, since the intensity function often becomes too steep after the observation period. In particular, the proposed priors allow decision maker to explicitly account for independence between λ_0 and β, and are a significant improvement over previous approaches, which have generally been based on the assumption either that β is

known or that λ_0 and β are independent. Furthermore, the prior distribution for the power law failure model has more advantages than the corresponding distribution for the exponential failure model, since it has a wide range of intensity function shapes.

Further work could relax the assumption that medication times can be neglected, that medication take place instantaneously after failure. Besides, the assumption that the decision maker is risk neutral can also be relaxed, by eliciting the loss functions in terms of utility instead of monetary values and hence considering the decision maker's risk attitude, the cost of a failure, the cost of the intervention treatment, the cost of collecting information, the lifetime, and the risk reduction fraction are known constants instead of random variables. However, in such case, simulation would be more appropriate than sensitivity analysis for studying the resulting model, since there would be too many random variables involved in the decision process.

Finally, risk modeling for aging chronic diseases requires the development of new concepts and methodologies. This is because important substantive issues arise in the analysis of aging chronic disease risks that do not arise in the context of modeling risk processes for acute diseases or disease episodes. Probably the most fundamental difference in modeling aging chronic diseases is that such diseases behave as processes, with their own internal time-dimension, rather than as shocks or instantaneous events. Hence a major purpose of Bayesian value of information analysis is to assist in comprehension of the problem and to give decision maker insight into what variables or features of the problem should have a major impact on the further research.

REFERENCES

Ades, A. E. 2002. Markov chain Monte Carlo estimation of a multi-parameter decision model: consistency of evidence, and the accurate assessment of uncertainty. *Medical Decision Making 22*, 359–371.

Albisser, A. M., Hodel, A. S., Albisser J. B., et al. 2002. Chronic disease management: a systems model relating outcomes, reporting, monitoring, interventions and satisfaction. *Control Engineering Practice 10*, 101–110.

Chang, C. C., Cheng, C. S., Huang, Y. S. 2006. A Web-Based Decision Support Systems for Chronic Disease. *Journal of Universal Computer Science 12*, 115–125.

Chang, C. C., Cheng, C. S. 2007. A Structural Design of Clinical Decision Support System for Chronic Diseases Risk Management. *Central European Journal of Medicine 2*, 2, 129–139.

Claxton, K., Thompson, K. 2001. A dynamic programming approach to efficient clinical trial design. *Journal of Health Economic 20*, 432–448.

Claxton, K. 1999. The irrelevance of inference: A decision making approach to the stochastic evaluation of health care technologies. *Journal of Health Economics 18*, 341–364.

Feltham, G. A. 1968. The value of information. *The Accounting Review 43*, 4, 684–696.

Fenwick, E., Claxton, K., Sculpher, M., et al. 2000. Improving the efficiency and relevance of health technology assessment: the role of decision analytic modeling. *Centre for Health Economics Discussion 179*, York: Centre for Health Economics.

Flockhart, D. E., Sham, C. H., Xiao, Y. 1993. Maximizing the value of information for ground water protection: three test cases. *Water Resources Bulletin: American Water Resources Association 29*, 6, 957–964.

Hu, P., Zelen, M. 1997. Planning Clinical Trials to Evaluate Early Detection Programmes. *Biometrika 84*, 4, 817–829.

International Chronic Granulomatous Disease Cooperative Study Group 1991. A controlled trial of interferon gamma to prevent infection in chronic Granulomatous disease. *New England Journal of Medicine 324*, 509–516.

KATSAHIAN, S., PORCHER, R., MARY, J. Y., ET AL 2004. The graft-versus-leukaemia effect after allogeneic bone-marrow transplantation: assessment through competing risks approaches. *Statistics in Medicine 23*, 3851–3863.

LAWLESS, J. F., NADEAU, J. C. 1995. Some simple robust methods for the analysis of recurrent events. *Technometrics 37*, 158–168.

MANTON, K. G., STALLARD, E. 1988. Chronic Disease Modelling: Measurement and Evaluation of the Risks of Chronic Disease Processes. New York, Oxford University Press.

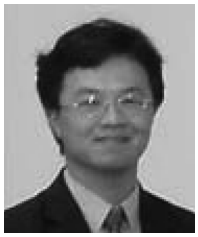
PRATT, J., RAIFFA, H., SCHLAIFER, R. 1995. Introduction to statistical decision theory. Cambridge, MA: MIT Press.

RAIFFA, H., SCHLAIFER, R. 1959. Probability and statistics for business decisions. New York: McGraw-Hill.

RAIFFA, H., SCHLAIFER, R. 1968. Applied Statistical Decision Theory. Cambridge, Massachusetts, Massachusetts Institute of Technology Press.

SENDI, P., GAFNI, A., BIRCH, S. 2002. Opportunity costs and uncertainty in the economic evaluation of health care interventions. *Health Economics 11*, 23–31.

THOMPSON, K. M., EVANS, J. S. 1997. The value of improved national exposure information for perchloroethylene (perc): a case study for dry cleaners. *Risk Analysis 17*, 253–271.



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